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FOUNDATIONS OF MODERN MACROECONOMICS

BEN J. HEIJDR

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In memoriam

Jenny E. Ligthart

November 6, 1967 – November 21, 2012

Walter H. Fisher

December 9, 1961 – November 15, 2012

Preface

What can the book do for you?

As was the case for the first two editions of this book, this new edition tries to present a balanced overview of modern macroeconomic theory. I follow two guiding principles. First, I adopt a rather eclectic approach by paying attention not just to the most recent insights in the field but also to developments that are currently less popular. In doing so, I hope to provide students with a better overview of current *and* past debates in macroeconomic theory. History can teach us useful lessons, provided we are willing to listen! For example, I continue to include discussions of the IS-LM model, the adaptive expectations hypothesis, and the Solow-Swan growth model (to mention a few). Though these theories are currently less fashionable (and, as some economists argue, may even be “outdated”), it is my firm conviction that they nevertheless provide important insights. For example, to fully appreciate the importance of the rational expectations hypothesis, a good understanding of the adaptive expectations hypothesis (its immediate predecessor) is indispensable. Similarly, to really understand the contributions made in recent years by New Keynesian Dynamic Stochastic General Equilibrium (DSGE) economists, it is very useful to have a firm understanding of the IS-LM model. Also, a good grasp of the Solow-Swan model helps in appreciating the Ramsey model and the endogenous growth models formulated in the 1980s and 1990s. Of course, as the saying goes, “old habits die slowly” and the IS-LM model is still used extensively even though, as Blanchard has pointed out, many people may not even know they are using it (2000, p. 1405).

The second guiding principle concerns the expositional style of the book. In addition to introducing the different theories by verbal and graphical means, I have also aimed to successively develop “the tools of the trade” of modern macroeconomics. In this aspect the book is related to Allen’s (1967) marvellous macroeconomic tool-book. So instead of only providing students with a verbal/intuitive understanding of the material (valuable as it is), I also explain the basic modelling tricks of modern macroeconomics. Where needed the full details of both the models and their solutions are presented. Students who have worked through the textbook (and its accompanying manual) should have little or no problems reading the recent journal literature in macroeconomics or building their own macro models.

How can the book be used?

Depending on the background of students, the book can be used in the undergraduate and/or the graduate curriculum. Part I, consisting of Chapters 1-9, can be used in an intermediate macroeconomics course in the undergraduate curriculum. For

example, I use Chapters 1-6 in my seven-week macroeconomics course in the third-year of the bachelor program at the University of Groningen. Economics students in this course have been exposed to Blanchard et al. (2013) in their first two years, whilst econometrics students have studied Gärtner (2016) in their second year of studies. In addition, these students have studied basic mathematical methods at the level discussed, for example, in Hoy et al. (2011).

Parts II and III of the book consist of Chapters 10-19. They are aimed at advanced bachelor students, first-year master students, and beginning doctoral students. In the graduate curriculum, the book can be used as the main text in a first-semester macroeconomics course or as a supplementary text for an advanced graduate macro course. At the University of Groningen, for example, I use Chapters 12-15 in my half-semester macroeconomics course in the regular masters programs. In the research master courses I also cover Chapter 10 and most of Chapters 16-19. The book is also well-suited for beginning doctoral students with no (or insufficient) previous training in macroeconomic theory. Parts of Chapters 12-16 were used in the various graduate courses I have taught over the years for the Netherlands Network of Economics (NAKE), the Tinbergen Institute, CESifo, and the Institute for Advanced Studies (Vienna).

Intermezzos

The book contains a number of so-called intermezzos. I use the term ‘intermezzo’ in an extended and unusual sense. Recall that in music an intermezzo is a composition that is played in between acts of a play or movements of a much larger musical piece. In this book, the intermezzos do not make any sound but, like in music, they are ‘small morsels in between big chunks’. They serve a number of purposes. First of all, they ensure that upon first reading students are not distracted by complex technical intricacies. Second, they allow for in-depth coverage of a number of key results in theoretical macroeconomics. Furthermore, in combination with the chapter appendices and the mathematical appendix at the end the book, they cover all technicalities necessary for a sound understanding of modern macroeconomics. Whereas the appendices are purely aimed at mathematical results, the intermezzos focus more on the fault line between mathematics and theoretical macroeconomics. Finally, the intermezzos serve as reference tools for readers who wish to reacquaint themselves with things they used to know but have forgotten.

Starred sections

In this edition I have also included sections marked with a superscript star (★). These sections contain material that is more difficult than the rest of the chapter in which they are located. Students may choose to skip the starred material when first reading the chapter. Upon completion of the book the successful student will find that most (or even all) stars have become invisible.

Changes for the Third Edition

The book has been thoroughly rewritten. Compared to the second edition, it has grown in size by about one hundred pages. The main changes are as follows.

- The current book includes forty-seven intermezzos, of which sixteen are new. All of these have been extensively checked and streamlined. They are numbered and carry an informative title. A List of Intermezzos is included in the preamble of the book which facilitates cross-referencing. The numbering system is as before, with the first digit denoting the chapter in which the intermezzo is located. The new intermezzos are 1.1, 1.2, 5.1, 5.2, 8.2, 8.3, 9.1, 12.2, 13.3, 16.1, 17.1, 18.1, 18.2, 19.1, 19.2 and 19.3.
- The new Chapter 2 deals exclusively with the open economy. It follows logically from the first chapter and contains material from Sections 1 and 2 of the old Chapter 10.
- Chapter 3 is a rewritten version of the old Chapter 2.
- Chapter 4 has been renamed to better reflect its contents. It contains a rewritten version of the old Chapter 4 as well as Section 3 (on the Dornbusch model) from the old Chapter 10.
- Chapter 5 is an expanded and rewritten version of the old Chapter 3. It now includes a small open economy model and explains the Dynare software package that can be used to solve rational expectations models.
- Chapter 6 is a lightly rewritten version of the old Chapter 5.
- Chapter 7 is a thoroughly edited and shortened version of the old Chapters 6 and 7. It also contains some new material on union- and efficiency-wage models in general equilibrium.
- Chapter 8 is an expanded version of the old Chapter 8. It now contains a section on endogenous job destruction.
- Chapter 9 has been renamed to better reflect its contents. In addition it has been expanded and now includes a discussion of dynamic inconsistency of individual choices resulting from present-biased (or quasi-hyperbolic) preferences.
- Chapters 10 and 11 are lightly edited versions of the old Chapters 11 and 12.
- The old Chapter 12 (on exogenous growth) has been split into two much expanded chapters. The new Chapter 12 deals exclusively with Solow-Swan style growth models. It has been expanded somewhat and now also features a section of the two-sector Meade-Uzawa model.
- Chapter 13 contains Sections 13.5–13.7 from the old Chapter 13. In addition it has been expanded dramatically. It now includes models with endogenous labour supply (using material from the old Chapter 15), search unemployment, and money balances entering the felicity function. This is the pivotal chapter in the book as the Ramsey-Cass-Koopmans model that it covers in all its guises plays a central role in the material that follows from there on.
- Chapter 14 is a lightly edited version of the old Chapter 14. Similarly, Chapters 15 and 16 are lightly edited versions of the old Chapters 16 and 17.
- Chapter 17 is brand new. It provides a brief (and mostly intuitive) discussion of the method of dynamic programming (DP). In addition it introduces the concept of complete markets and shows how one can construct a “representative

agent” in such a setting. Whilst a deep knowledge of DP is not really essential to understand Chapters 18–19, it is indispensable if one wants to proceed to the more advanced literature in macroeconomics, e.g. the graduate textbook by Ljungqvist and Sargent (2012).

- Chapter 18 is the first chapter on the DSGE approach. It contains material from the old Section 15.5. It has been edited thoroughly and now includes discussions of the stochastic discount factor and shows how DSGE models can be simulated with the aid of the Dynare software package (introduced in Chapter 5).
- Chapter 19 is brand new. It contains a thorough discussion of the New Keynesian DSGE approach and finishes with a brief assessment of the state of the art at the time of writing. This assessment replaces the Epilogue from the second edition.

Visible means of support

It somehow seems impossible to produce a book of this size without generating (free of charge) some typos and errors. Needless to say, all such errors and typos will be published as I become aware of them. I will make the errata documents available through the website for the book:

<http://www.heijdra.org/fomm3>

So please let me know about any typos and/or errors that you may spot. This is what you can do for the book! The contact address is: info@heijdra.org. As a (weak substitute for a) reward, I will mention your name prominently on the website (as having contributed to the public good). Of course, your name will also feature in the Acknowledgements section in any future edition of the book.

The website also includes ready-to-use slides for all chapters in PDF format. Teachers who wish to adapt these slides to their own purpose or software platform can download the $\text{\LaTeX}2_{\epsilon}$ code and all figures (in EPS and EMF formats) and proceed from there.

I have updated and streamlined the accompanying *Exercise and Solutions Manual* which is published by Oxford University Press. This hands-on exercise book contains a large number of problems plus model answers. These problem sets allow the interested student to further develop his/her skills.

Acknowledgements

In preparing the *third edition*, I received useful comments from many people, including Pieter Ijtsma, Gerard van der Meijden, Laurie Reijnders, Girum Dagnachew Abate, Wilma Huitema, Christien de Kort, Stine Celius, Kengo Tahara, Carolien Calkhoven, Mika Kortelainen, Matthijs Katz, Yoni Schirris, Bastiaan Quast, Jelle van Essen, Marc Boom, Annemarije Santman, Bart Rutjes, Lisan Spiegelaar, Gert-Jan Romensen, Jitka Vavra, and Vesa-Matti Heikkuri. One of the great privileges of working in a university is that – surrounded by young and enthusiastic people – one never really grows old. Life-long learning is the norm rather than the exception in academia. Over the years I have greatly benefited from the interaction with

some outstanding colleagues. The collaboration with Fabian Kindermann (University of Bonn) has resulted in a significant upgrade of my computing skills – something that was long overdue. Since he is a great teacher of computational economics, I highly recommend his forthcoming textbook on Fortran computing (Fehr and Kindermann, 2017). My friend and colleague, Pim Heijnen (University of Groningen) not only accompanied me to the pub quite regularly (for work-related meetings) but also had a very significant effect on my computing skills. In addition, he suggested the cake-eating example of dynamic inconsistency (and hyperbolic discounting) that is discussed in Chapter 9. Another colleague, Allard van der Made, has been a great sounding board on mathematical issues. I also thank my Groningen colleagues Lammertjan Dam for discussions on the consumption-based asset pricing model employed in Chapters 18–19, Christiaan van der Kwaak for comments on Chapters 17 and 19, and Gerard Kuper for plowing through Chapter 18. Just as for the second edition, Jochen Mierau has read the entire manuscript and has provided useful advice on many aspects of the book (both content and exposition). To the extent that I have not followed some (or all) of their suggestions, this is not because I disagreed with them but rather because of a binding time constraint on my part.

As with the previous two editions of the book, Siep Kroonenberg has assisted at crucial instances with the more complicated aspects of the $\text{\LaTeX}2_{\epsilon}$ codes used to produce this book. Of course, Leslie Lamport and Donald E. Knuth are thanked implicitly too for producing, respectively, $\text{\LaTeX}2_{\epsilon}$ and \TeX .

Over the years the following people from Oxford University Press have been of great assistance in the production and marketing of this book: Andrew Schuller, Rebecca Bryant, Sarah Dobson, Jennifer Wilkinson, Sarah Caro, Aimee Wright, T.W. Bartel, and Adam Swallow. During the fine-tuning of the book I benefited tremendously from the efforts of Katie Bishop, Elisa Cozzi, and Joshua Hey. I thank all of them for their efforts.

Dedication

I dedicate this new edition of the book to Jenny E. Ligthart and Walter H. Fisher who passed away during a *hebdomas horribilis* in November 2012. I first met Jenny in December 1992. I moved to the University of Amsterdam (where she was a Ph.D. student) in May 1993 and together with Rick van der Ploeg I ended up supervising Jenny’s doctoral work. Following her successful thesis defence in November 1997 we continued to work together on various projects until her passing. I first met Walter in May 2002 when he invited me to the Institute for Advanced Studies (IHS) in Vienna. From 2006-2013 I held a Visiting Research Professorship at the institute and our collaboration became much more intensive. Jenny and Walter were much more than nice colleagues and co-authors to me. Over time they became close personal friends. Some of their work finds its way into this book. The loyalty and friendship I received from them does not. I will cherish their memory for as long as I live.

Ben J. Heijdra

University of Groningen, The Netherlands
February 2017

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Part I

Intermediate macroeconomics

Chapter 1

Review of the AD-AS model

The purpose of this chapter is to achieve three goals:

1. To (partially) refresh and extend the macroeconomic knowledge from first-year courses.
2. To investigate the effectiveness of monetary and fiscal policy on output, employment, the interest rate, and the price level.
3. To introduce the most important past and current schools of thought in macroeconomics.

In order to achieve these goals, we first have to discuss some elementary concepts relating to the *aggregate labour market* and the *demand for money*. It turns out that the most important differences of opinion between (most varieties of) Classical and Keynesian economists can be traced back to their respective assumptions regarding the labour market, expectation formation, and money demand.

1.1 The aggregate labour market

Our discussion of the labour market in this chapter is very basic. In Chapters 7–8 we return to this important topic in more detail. The stylized account of the labour market uses the devices of the aggregate demand for and supply of labour.

1.1.1 The demand for labour

The central element in the basic theory of labour demand is the production function. Perfectly competitive profit-maximizing entrepreneurs utilize this production function under the restriction that the capital stock is given in the short run. The production function is thus given by:

$$Y = F(N, \bar{K}), \tag{1.1}$$

where Y is real output, \bar{K} is the given capital stock (machines, PCs, cars), N is the amount of labour employed, and $F(N, \bar{K})$ is the production function. The marginal products of labour and capital are denoted by $F_N \equiv \partial F(N, \bar{K}) / \partial N$ and $F_K \equiv \partial F(N, \bar{K}) / \partial \bar{K}$, respectively. Furthermore, we assume that the marginal product of labour (capital) declines as employment (capital) is increased, i.e. $F_{NN} \equiv \partial^2 F(N, \bar{K}) /$

$\partial N^2 < 0$ ($F_{KK} \equiv \partial^2 F(N, \bar{K}) / \partial \bar{K}^2 < 0$). Too many cooks in the kitchen spoil the broth. We also assume that the factors are cooperative in the sense that increasing one factor raises the marginal productivity of the other factor ($\partial^2 F(N, \bar{K}) / \partial \bar{K} \partial N \equiv F_{KN} = F_{NK} \equiv \partial^2 F(N, \bar{K}) / \partial N \partial \bar{K} > 0$). The use of robot mixers in the kitchen thus enhances the productivity of the cooks. Finally, we assume constant returns to scale so that doubling all factors of production induces a doubling of output. More precisely, $F(\lambda N, \lambda \bar{K}) = \lambda F(N, \bar{K})$ with λ any positive constant.

Short-run profits are defined as revenues minus the wage bill:

$$\Pi \equiv PY - WN, \quad (1.2)$$

where Π is nominal profit, P is the price charged by the firm, and W is the nominal wage rate. In words, all revenue (PY) that is not paid to the variable production factor labour in terms of wages (WN) is considered profit, which is the reward that accrues to the owners of the capital stock (note that we ignore taxes for the moment).

We assume perfect competition on the aggregate goods market, so that the individual firm cannot exert any influence on the price it charges for its product. Hence, the only choice that is open to the firm (in the short run) is to determine the amount of production (Y) and employment (N) such that profit is maximized. By substituting the production function in the profit definition, we see that once employment is chosen, output is also automatically chosen. The problem for the firm is thus to choose N to maximize Π :

$$\max_{\{N\}} \Pi \equiv PF(N, \bar{K}) - WN. \quad (1.3)$$

The firm can do no better than to follow the following decision rule:

$$\frac{d\Pi}{dN} = 0: \quad PF_N(N, \bar{K}) - W = 0, \quad (1.4)$$

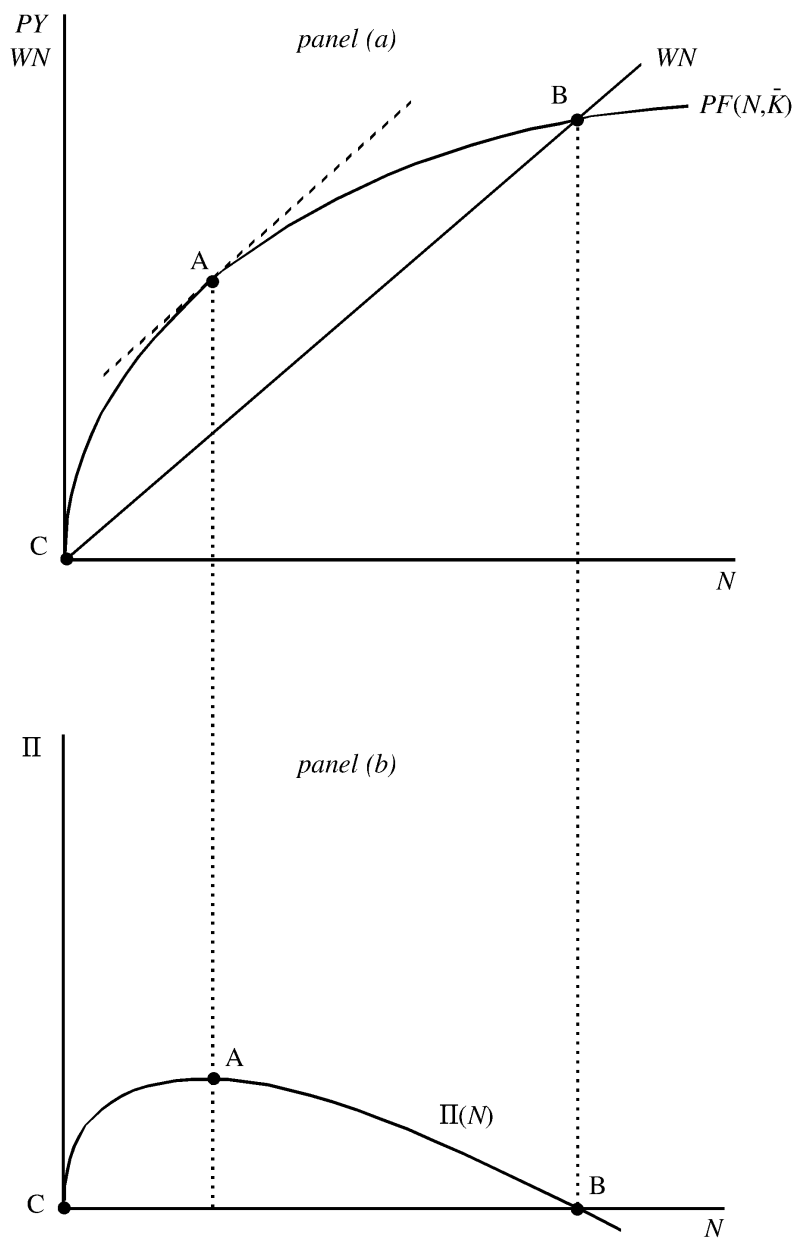
where the second-order condition implies that (1.4) describes a maximum: $d^2\Pi/dN^2 = PF_{NN} < 0$ (because $P > 0$ and $F_{NN} < 0$ by assumption). The interpretation of (1.4) is clear; the firm should keep expanding its employment up to the point where the marginal unit of labour exactly breaks even (in the sense that the additional output produced by the marginal worker yields a revenue that exactly covers the wage that is paid to the worker). In terms of Figure 1.1, the profit maximum occurs at point A. (At points B and C the firm makes no profits.)

The decision rule (1.4) is a vitally important element in the macroeconomic labour market story. It is also relatively uncontroversial: virtually all macroeconomists believe in some version of equation (1.4). We can easily transform (1.4) into the demand for labour, a schedule which shows how much labour a firm wants to hire for a given real wage rate. Formally, we can view equation (1.4) as an implicit relationship between N^D (the superscript “D” stands for demand) on the one hand and the real wage, W/P , and the given capital stock, \bar{K} , on the other. The partial derivatives of this implicit relationship can be obtained by using the trick of implicit functions. First, we totally differentiate equation (1.4):

$$dF_N(N^D, \bar{K}) = d(W/P) \quad \Rightarrow \quad F_{NN}dN^D + F_{NK}d\bar{K} = d(W/P), \quad (1.5)$$

or, after rearranging terms:

$$dN^D = -\frac{F_{NK}}{F_{NN}}d\bar{K} + \frac{1}{F_{NN}}d(W/P). \quad (1.6)$$

**Figure 1.1: Short-run profit maximization**

Since $F_{NN} < 0$, the marginal product of labour falls as more units of labour are employed. As a result, equation (1.6) states that a higher real wage ($d(W/P) > 0$) diminishes the demand for labour ($dN^D < 0$) ceteris paribus (i.e. holding \bar{K} constant). Hence, $1/F_{NN}$ in equation (1.6) can be interpreted as the partial derivative of the implicit function between N^D and $(W/P, \bar{K})$ with respect to the real wage, W/P .

The partial derivative with respect to the capital stock is obtained in a similar fashion (and is equal to $-F_{NK}/F_{NN} > 0$). Since labour and capital are cooperative factors of production, increasing the capital stock raises the marginal product of labour. For a given real wage rate, the profit-maximizing firm thus hires more labour.

In summary, we can write:

$$N^D = N^D(W/P, \bar{K}) \quad N_{W/P}^D \equiv \frac{1}{F_{NN}} < 0, \quad N_{\bar{K}}^D \equiv -\frac{F_{NK}}{F_{NN}} > 0. \quad (1.7)$$

In terms of Figure 1.2, varying the real wage rate implies a movement along a given demand for labour curve, whilst increasing the capital stock shifts the demand curve to the right. A higher cost of labour or a lower capital stock necessitates a higher marginal productivity of labour and thus a lower demand for labour.

Intermezzo 1.1

The Cobb-Douglas production function and labour demand. In this intermezzo we discuss an often-used two-factor production function featuring constant returns to scale. The Cobb-Douglas function can be written as:

$$F(N, K) \equiv Z_0 K^\alpha N^{1-\alpha}, \quad 0 < \alpha < 1, \quad (a)$$

where α is an efficiency parameter and Z_0 is a scaling factor. Several things are worth noting. First, it is easy to verify that this function features constant returns to scale:

$$F(\lambda N, \lambda K) = Z_0 (\lambda K)^\alpha (\lambda N)^{1-\alpha} = \lambda^{\alpha+1-\alpha} Z_0 K^\alpha N^{1-\alpha} = \lambda F(N, K). \quad (b)$$

Second, the marginal products of labour and capital are both positive:

$$F_N(N, K) = (1 - \alpha) Z_0 \left(\frac{K}{N}\right)^\alpha > 0, \quad (c)$$

$$F_K(N, K) = \alpha Z_0 \left(\frac{K}{N}\right)^{-(1-\alpha)} > 0. \quad (d)$$

Third, each factor features diminishing marginal productivity, and the factors are cooperative:

$$F_{NN}(N, K) = -\alpha(1 - \alpha) Z_0 \left(\frac{K}{N}\right)^\alpha \frac{1}{N} < 0, \quad (e)$$

$$F_{KK}(N, K) = -\alpha(1 - \alpha) Z_0 \left(\frac{K}{N}\right)^{-(1-\alpha)} \frac{1}{K} < 0, \quad (f)$$

$$F_{NK}(N, K) = \alpha(1 - \alpha) Z_0 \left(\frac{K}{N}\right)^\alpha \frac{1}{K} > 0. \quad (g)$$

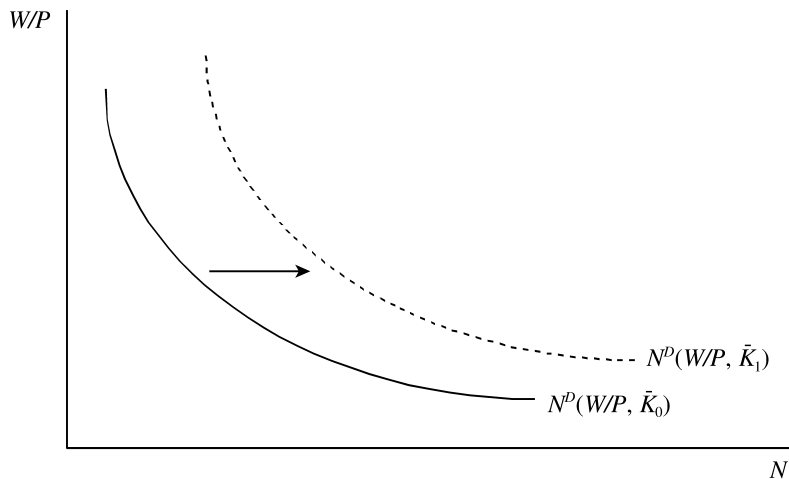


Figure 1.2: The demand for labour

Fourth, for a given capital stock \bar{K} the competitive labour demand function can be written as:

$$N^D = \left(\frac{(1 - \alpha)Z_0}{w} \right)^{1/\alpha} \bar{K}, \quad (\text{h})$$

where $w \equiv W/P$ is the real wage rate. The (absolute value of the) wage elasticity of labour demand—defined in general as $\varepsilon_D \equiv -F_N(N, \bar{K}) / (NF_{NN}(N, \bar{K}))$ —thus equals $\varepsilon_D = 1/\alpha$ for the Cobb-Douglas production function. Fifth, provided labour is paid its marginal product, the labour income share in production is constant, i.e. $wN/F(N, K) = 1 - \alpha$.

1.1.2 The supply of labour

In the previous section we implicitly assumed that firms can freely observe the actual values of the price level and the wage rate (P and W). This is realistic enough, because all the individual firm must do is to observe its *own* price and the wage paid to its *own* workers.

Matters are somewhat more complicated for the households, who are the suppliers of labour in our stylized account of the labour market. Indeed, in the decision about goods consumption and labour supply, the households may know their own nominal wage (W) with certainty, but they may not know how much they can actually consume with that wage. The household has to estimate the price of a whole basket of goods, a task inherently more difficult than the one facing the individual firm. The simplest way to introduce this asymmetry in information is to assume that the household forms a guess about the aggregate price level, denoted by P^e (where the superscript “ e ” stands for expected).

The household derives utility from goods consumption (denoted by C) and leisure ($1 - N^S$). The household “owns” one unit of time, of which N^S units are spent working, so that time available for leisure is equal to $1 - N^S$. We write the utility function in general terms as $U(C, 1 - N^S)$ and assume positive but diminishing marginal utilities: $U_C > 0$, $U_{1-N} > 0$, $U_{CC} < 0$, and $U_{1-N,1-N} < 0$. Some extra consumption of goods and leisure is fun, but less so if you already consume a lot or have plenty of spare time to enjoy. In addition, we assume that indifference curves bulge toward the origin, i.e. $U_{CC}U_{1-N,1-N} - U_{C,1-N}^2 > 0$.

The household chooses that combination of C and $1 - N^S$ for which the highest possible satisfaction is attained (as measured by $U(\cdot, \cdot)$), given the expected price level, P^e , and the (expected) budget restriction $P^e C = WN^S$. We assume that the household has no sources of income other than wages. Formally, we can thus write the problem for the household as follows:

$$\max_{\{C, N^S\}} U \equiv U(C, 1 - N^S) \quad \text{subject to} \quad P^e C = WN^S. \quad (1.8)$$

This problem looks rather prohibitive, but we can make it easier by substituting the level of consumption implied by the budget restriction ($C = (W/P^e)N^S$) into the utility function. The household then only has to choose the level of labour supply:

$$\max_{\{N^S\}} U \equiv U\left(\left(\frac{W}{P^e}\right)N^S, 1 - N^S\right). \quad (1.9)$$

This yields a straightforward decision rule for the household:

$$\frac{dU}{dN^S} = 0: \quad \left(\frac{W}{P^e}\right)U_C - U_{1-N} = 0. \quad (1.10)$$

The first term on the left-hand side (i.e. $(W/P^e)U_C$) measures the marginal benefit of supplying one extra unit of labour to the labour market. By working more, the household obtains more income, especially if the real wage is high, and hence more consumption. The second term (i.e. U_{1-N}) measures the marginal cost of that extra unit. By supplying more labour, the household misses out on valuable leisure time. In an optimum the household sets the marginal benefit equal to the marginal cost of supplying an additional unit of labour.

In principle we could now proceed by investigating what happens to labour supply and consumption if the expected real wage rate is varied. Mathematically this is slightly more involved than for the labour demand equation, so that we first derive the basic intuition concerning labour supply by graphical means. (The mathematical derivation of labour supply is given in Chapter 7.)

In Figure 1.3 we plot consumption on the vertical axis and leisure on the horizontal axis. The initial expected real wage is $(W/P^e)_0$, and the budget line goes through \bar{C}_0 ($\equiv (W/P^e)_0$) on the C -axis, and 1 on the $(1 - N^S)$ -axis. The optimal consumption-leisure choice occurs at the point where an indifference curve has a tangency with the budget line. This occurs at point E_0 , where consumption is C_0 , leisure is $1 - N_0^S$, and the level of utility is U_0 . By plotting the implied value of labour supply, N_0^S , against the expected real wage rate in Figure 1.4, we obtain the first point on the labour supply curve.

Suppose now that the expected real wage is a bit higher, say $(W/P^e)_1$. In terms of Figure 1.3 this implies that the budget line rotates in a clockwise fashion around the intersection point on the leisure axis. The new intersection on the consumption axis is at \bar{C}_1 ($\equiv (W/P^e)_1$). For the case drawn, the new optimum choice occurs at

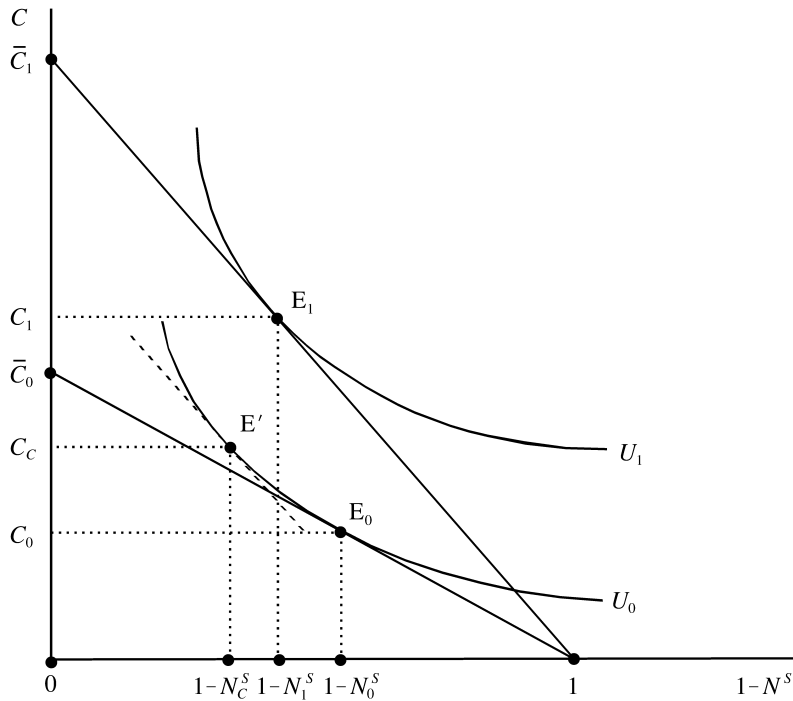


Figure 1.3: The consumption-leisure choice

point E_1 , which lies above and to the left of the initial point E_0 . Consumption is C_1 , leisure is $1 - N_1^S$, and the level of utility is U_1 . By plotting the implied value of labour supply, N_1^S , against the real wage rate in Figure 1.4, we obtain the second point on the labour supply curve. By connecting the two points we obtain the labour supply schedule, labelled $N^S(W/P^e)$, which for the case drawn slopes upward.

Unlike the labour demand curve, which always slopes downwards, the slope of the labour supply curve is not necessarily positive. The reason is that there are two, potentially offsetting, effects that confront the household when the expected real wage rises. The first effect is called the *pure substitution effect*. To determine this effect, we ask ourselves what combination of consumption and leisure the household would choose at the higher expected real wage if it were somehow restricted to remain at the initial level of utility U_0 . In Figure 1.3, we see that the household would choose point E' , where consumption is C_c , leisure is $1 - N_c^S$, and labour supply is N_c^S (the subscript “C” stands for compensated). The move from the initial point E_0 to the (hypothetical) compensated point E' constitutes the pure substitution effect (i.e. *SE*). Intuitively, the pure substitution effect says that a household will buy less of anything for which the price has risen. A rise in the expected real wage rate means that the price of leisure has gone up. Consequently, the household buys less of it. This gives us an interesting result: *the compensated labour supply curve is always upward sloping* (see $N^S(W/P^e, U_0)$ in Figure 1.4).

The second effect is called the *income effect*. It says that, for a given initial level of labour supply N_0^S , a higher expected real wage implies a higher expected real income, or, $(W/P^e)_1 N_0^S > (W/P^e)_0 N_0^S$. Provided leisure is a normal good the household would react to this higher income by purchasing *more* leisure, not less. Hence, the income effect (i.e. *IE*), which is represented by the move from point E' to E_1 ,